



Reg. No. : .....

Name : .....

**Third Semester B.Tech. Degree Examination, January 2016**  
**(2013 Scheme)**  
**13.303 : DISCRETE STRUCTURES (FR)**

Time : 3 Hours

Max. Marks : 100

**PART – A**



Answer **all** questions. **Each** question carries **2** marks.

1. There are two restaurants next to each other. One has a board which says "Good food is not cheap", and the other has another board that says "Cheap food is not good". Are the two boards saying the same thing ? Prove your answer.
2. Prove the following implication  $(P \wedge Q) \Rightarrow (P \rightarrow Q)$ .
3. If  $\{\{a, c, e\}, \{b, d, f\}\}$  is a partition of the set  $\{a, b, c, d, e, f\}$ , determine the corresponding equivalence relation  $R$ .
4. If  $P = \{2, 3, 6, 12, 24, 36\}$ . Let ' $\leq$ ' be the relation defined on the set  $P$  such that  $x \leq y$  if  $x$  divides  $y$ . Draw the Hasse diagram of  $(P, \leq)$ .
5. Explain Cantor's theorem of power sets.
6. Symbolic the expression "x is the father of the mother of y".
7. What are permutation groups ?
8. Prove that the inverse of an element in a group is unique.
9. What is a distributive lattice ?
10. Explain the connectivity of a graph.



## PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

## Module – I

11. a) Determine the validity of the following arguments

$$A \Rightarrow (B \vee C), D \Rightarrow \neg C, B \Rightarrow \neg A, A, D.$$

$$\text{Conclusion : } B \wedge \neg B$$

10

b) Show that  $\neg P \vee Q, \sim Q \vee R, R \rightarrow S \Rightarrow P \rightarrow S.$

10

OR

12. a) Show that  $\forall x (P(x) \vee Q(x)) \Rightarrow \forall x P(x) \vee \exists x Q(x).$

10

b) Show that  $S \vee R$  is tautologically implied by  $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S).$

5

c) Show the following implication using rules of inferences

$$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S).$$

5

## Module – II

13. a) The following function is defined on the set of integers.

$$f(k) = k + 1, g(k) = 2k, h(k) = 3k$$

i) Which of these functions are one-one ?

ii) Which of these functions are onto ?

iii) Find  $f \circ g$  and  $g \circ f.$

8

b) Let  $X = \{1, 2, \dots, 7\}$  and  $R = \{\langle x, y \rangle / x - y \text{ is divisible by } 3\}.$  Show that  $R$  is an equivalence relation.

6

c) Prove by induction that  $(1 + 2 + 3 + \dots + n)^2 = 1^3 + 2^3 + \dots + n^3$  for all +ve integers.

6

OR

14. a) For any two sets  $A$  and  $B$ , show that  $A - (A \cap B) = A - B.$

5

b) Five friends run a race everyday for 4 month (excluding feb). If no race ends in a to, show that there are atleast 2 races with identical outcomes.

5

c) What are Piano Axioms ? Explain.

5

d) Let  $a_0 = 1, a_1 = 2, a_2 = 3, a_n = a_{n-1} + a_{n-2} + a_{n-3}$  for  $n \geq 3.$  Prove that  $a_n \leq 3^n.$

5



**Module – III**

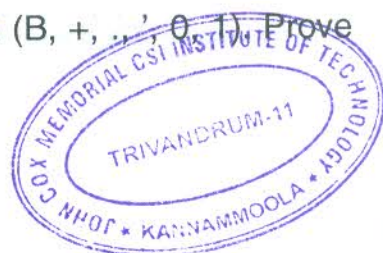
- 15. a) If  $(a + b)^2 = a^2 + 2ab + b^2 \forall a, b \in R$ , prove that R is a commutative ring and conversely. 5
- b) Prove that the set Q of all rational numbers other than with the operations defined by  $a * b = a + b - ab$  constitute an abelian group. 10
- c) Prove that the subgroup of a cyclic group is cyclic. 5

OR

- 16. a) Prove that the set  $(Z, *)$  of all integers for the composition of  $a * b = a + b + 1$  is an abelian group. 10
- b) Discuss algebraic system ring, integral domain and field with examples. 10

**Module – IV**

- 17. a) Let x, y be arbitrary elements in a Boolean algebra  $(B, +, \cdot, ', 0, 1)$ . Prove DeMorgan's laws.  
 $(x + y)' = x'y'$   
 $(xy)' = x' + y'$  10
- b) Show that the poset  $(Z^+, /)$ , the set of all positive integers under the relation of divisibility is a lattice. 5
- c) Differentiate between complete graph and connected graph with examples. 5



OR

- 18. a) State and prove any four basic properties of algebraic systems defined by lattices. 10
- b) Show that  $a \vee b$  is the least upper bound of a and b in  $(A, \leq)$ . Show that  $a \wedge b$  is the greatest lower bound of a and b in  $(A, \leq)$ . 5
- c) Differentiate between a boolean function and boolean expression. 5