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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, January 2016
(2013 Scheme)
13.303 : DISCRETE STRUCTURES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. There are two restaurants next to each other. One has a board which says "Good food is not cheap", and the other has another board that says "Cheap food is not good". Are the two boards saying the some thing ? Prove your answer.

2. Prove the following implication $(P \wedge Q) \Rightarrow (P \rightarrow Q)$.

3. If {{a, c, e}, {b, d, f} is a partition of the set {a, b, c, d, e, f}, determine the corresponding equivalence relation R.

4. If P = {2, 3, 6, 12, 24, 36}. Let '$\leq$' be the relation defined on the set P such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(P, \leq)$.

5. Explain Cantor's theorem of power sets.

6. Symbolic the expression "x is the father of the mother of y".

7. What are permutation groups ?

8. Prove that the inverse of an element in a group is unique.

9. What is a distributive lattice ?

10. Explain the connectivity of a graph.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) Determine the validity of the following arguments

$A \Rightarrow (B \vee C), D \Rightarrow \neg C, B \Rightarrow \neg A, A, D$.

Conclusion : $B \wedge \neg B$ **10**

b) Show that $\neg P \vee Q, \sim Q \vee R, R \rightarrow S \Rightarrow P \rightarrow S$. **10**

OR

12. a) Show that $\forall x\,(P(x) \vee Q(x)) \Rightarrow \forall x\,P(x) \vee \exists x\,Q(x)$. **10**

b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. **5**

c) Show the following implication using rules of inferences

$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$. **5**

Module – II

13. a) The following function is defined on the set of integers.

$f(k) = k + 1$, $g(k) = 2k$, $h(k) = 3k$

i) Which of there functions are one-one ?

ii) Which of there functions are onto ?

iii) Find $f_o g$ and $g_o f$. **8**

b) Let $X = \{1, 2, ..., 7\}$ and $R = \{\langle x, y\rangle / x - y$ is divisible by 3$\}$. Show that R is an equivalence relation. **6**

c) Prove by induction that $(1 + 2 + 3 + ... n)^2 = 1^3 + 2^3 + ... n^3$ for all +ve integers. **6**

OR

14. a) For any two sets A and B, show that $A - (A \cap B) = A - B$. **5**

b) Five friends run a race everyday for 4 month (excluding feb). If no race ends in a to, show that there are atleast 2 races with identical outcomes. **5**

c) What are Piano Axioms ? Explain. **5**

d) Let $a_0 = 1, a_1 = 2, a_2 = 3, a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. Prove that $a_n \leq 3^n$. **5**

Module – III

15. a) If $(a+b)^2 = a^2 + 2ab + b^2 \; \forall \; a, b \in R$, prove that R is a commutative ring and conversely. **5**

    b) Prove that the set Q of all rational numbers other than with the operations defined by $a * b = a + b - ab$ constitute an abelian group. **10**

    c) Prove that the subgroup of a cyclic group is cyclic. **5**

OR

16. a) Prove that the set $(Z, *)$ of all integers for the composition of $a * b = a + b + 1$ is an abelian group. **10**

    b) Discuss algebraic system ring, integral domain and field with examples. **10**

Module – IV

17. a) Let x, y be arbitrary elements in a Boolean algebra (B, +, ., ', 0, 1). Prove DeMorgan's laws.

    $(x + y)' = x'y'$

    $(xy)' = x' + y'$. **10**

    b) Show that the poset $(Z^+, /)$, the set of all positive integers under the relation of divisibility is a lattice. **5**

    c) Differentiate between complete graph and connected graph with examples. **5**

OR

18. a) State and prove any four basic properties of algebraic systems defined by lattices. **10**

    b) Show that $a \vee b$ is the least upper bound of a and b in $(A, \leq)$. Show that $a \wedge b$ is the greatest lower bound of a and b in $(A, \leq)$. **5**

    c) Differentiate between a boolean function and boolean expression. **5**
